

# Surface magnetic canting in a nonuniform film<sup>☆</sup>

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## Abstract

The zero temperature equilibrium configuration of a nonuniform system made of a ferromagnetic (FM) monolayer on top of a semi-infinite FM film is calculated using a nonlinear mapping formulation of mean-field theory, where the surface is taken into account via an appropriate boundary condition. The analytical criterion for the existence of surface magnetic canting, previously obtained by Popov and Pappas, is also recovered.

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Recently, nonuniform systems consisting of an ultra-thin magnetic film grown on top of a bulk-like film (e.g., 1.5 monolayers of Fe on a 15 nm-thick Gd(0001) film [1]) have attracted considerable interest since they may present, for opportune values of the surface and bulk anisotropies [2], a peculiar two-step reorientation transition [1]: a continuous one from in-plane to canted out-of-plane direction at low temperature, followed by a thermally irreversible rotation from canted to nearly perpendicular at higher temperature. At  $T = 0$ , the phase diagram of a uniform semi-infinite ferromagnet with competing surface and bulk uniaxial anisotropies was investigated by Popov and Pappas [3] for a magnetic field parallel to the film plane. Using a different mapping formulation [4] of mean-field theory, Betti et al. [5] studied the case of a perpendicular field. In this paper, we aim to generalize the map method [4] to the

nonuniform case in order to determine the  $T = 0$  equilibrium configurations of a system made of a surface magnetic monolayer (with saturation magnetization  $M_S$ , exchange constant  $J_S > 0$ , and anisotropy  $K_S < 0$ ) grown on top of a semi-infinite film (with magnetization  $M_B$ , exchange constant  $J_B > 0$ , and anisotropy  $K_B > 0$ ). The  $T = 0$  energy is

$$E = K_S M_S^2 \sin^2 \theta_1 - J_{SB} M_S M_B \cos(\theta_1 - \theta_2) + M_B^2 \sum_{n=2}^{\infty} [K_B \sin^2 \theta_n - J_B \cos(\theta_n - \theta_{n+1})], \quad (1)$$

where  $J_{SB} > 0$  is the interface exchange constant and  $\theta_n$  the angle formed by the classical vector moment of the  $n$ th layer with the film plane. The competing surface ( $K_S < 0$ ) and bulk ( $K_B > 0$ ) anisotropies favour respectively  $\theta = \pi/2$  and  $\theta = 0$ . In the framework of the map method [4], the  $T = 0$  equilibrium configurations of the system are obtained by  $\theta_n$ -derivation ( $n = 1, 2, \dots, \infty$ ) of the system energy, Eq. (1). Introducing the parameters  $\gamma = \frac{J_{SB} M_S M_B}{J_B M_B^2}$ ,  $\kappa_S = \frac{2K_S M_S^2}{J_{SB} M_S M_B}$  and  $\kappa_B = \frac{2K_B M_B^2}{J_B M_B^2}$ , the map equations can be written as

$$2s_2 = \kappa_S \sin(2\theta_1), \quad n = 1$$

$$2s_3 = 2\gamma s_2 + \kappa_B \sin(2\theta_2), \quad n = 2$$

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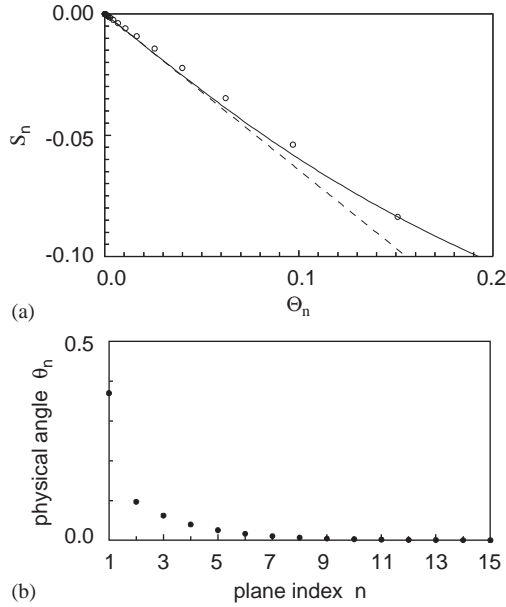


Fig. 1. (a) Map phase portrait, in the  $(\Theta, S)$  space, of a nonuniform film with  $\gamma = 0.2$ ,  $\kappa_B = 0.2$ ,  $\kappa_S = -0.8$ . Open circles:  $T = 0$  configuration, Eqs. (2), (3); full line: surface boundary condition (sbc); dashed line: tangent of sbc at the hyperbolic fixed point  $(\Theta_\infty, S_\infty) = (0, 0)$ , defined by Eq. (4). (b)  $T = 0$  magnetization profile.

$$2s_{n+1} = 2s_n + \kappa_B \sin(2\theta_n), \quad n \geq 3 \quad (2)$$

$$\theta_{n+1} = \theta_n + \sin^{-1}(s_{n+1}), \quad n \geq 1, \quad (3)$$

where  $s_n = \sin(\theta_n - \theta_{n-1})$ . By linearizing the map in the neighborhood of the hyperbolic fixed point  $(\theta_\infty, s_\infty) = (0, 0)$ , which corresponds to the bulk configuration [4], one obtains an eigenvalue equation with two real solutions:  $\lambda_1 = \exp(-\phi_0) < 1$  and  $\lambda_2 = \exp(\phi_0) > 1$ , where  $\cosh \phi_0 = 1 + \kappa_B/2$ . Now we observe that by making the transformation  $S_2 = \gamma s_2$ , the map equations for  $n = 2$  assume a bulk-like form. This can be accomplished even for  $n = 1$  by introducing the transformation  $\Theta_1 = \theta_1 + \sin^{-1}(S_2/\gamma) - \sin^{-1}(S_2)$  and by

requiring the vanishing of the quantity  $\gamma\kappa_S \sin(2\theta_1) - 2S_1 - \kappa_B \sin(2\theta_1)$ . In this way we obtain the surface boundary condition, shown in Fig. 1a as a full line, with equation  $\gamma\kappa_S \sin(2(\Theta_1 + \sin^{-1}(2S_1 + \kappa_B \sin(2\Theta_1)/2) - \sin^{-1}(2S_1 + \kappa_B \sin(2\Theta_1)/2\gamma))) - (2S_1 + \kappa_B \sin(2\Theta_1)) = 0$ . By virtue of the transformation to  $(\Theta, S)$  variables that restores map uniformity, the criterion for surface magnetic canting (SMC) in the nonuniform film is (see Fig. 1a)  $m_S \leq m_1$ , where  $m_1 = \lambda_1 - (\kappa_B + 1)$  is the slope of the orbit inflowing to the hyperbolic fixed point of the map and  $m_S$  is the slope of the surface boundary condition curve at the same point

$$m_S = \frac{\gamma\kappa_S(1 + \kappa_B) - \kappa_B(1 + \kappa_S)}{1 + (1 - \gamma)\kappa_S}. \quad (4)$$

The SMC condition can be rewritten as  $\kappa_S \leq (\kappa_S)_{\text{thr}} = -1 + \frac{\gamma\lambda_1}{1 - (1 - \gamma)\lambda_1}$ . Solving with respect to  $\kappa_B$ , we obtain

$$\kappa_B \leq -\frac{\kappa_S + 1}{(\gamma - 1)\kappa_S - 1} - 2 - \frac{(\gamma - 1)\kappa_S - 1}{\kappa_S + 1}, \quad (5)$$

that coincides with Eq. (42) in Ref. [3] in the case  $-1 < \kappa_S < 0$ , while SMC takes place at any magnitude of  $\kappa_B$  for  $\kappa_S < -1$ . In Fig. 1a we present the map phase portrait  $S(\Theta)$  calculated for  $\kappa_S = -0.8$  (i.e., just below the threshold  $(\kappa_S)_{\text{thr}} = -0.736$ ), and in Fig. 1b the corresponding  $T = 0$  magnetization profile (i.e., the physical angle  $\theta_n$  as a function of plane index). In conclusion, we have shown that using the nonlinear mapping method of Ref. [4] generalized to a nonuniform film, the analytical condition for SMC [3] is recovered and, in addition, the magnetization profile is easily calculated.

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